A disaggregated approach to understanding the connection between transactions and housing prices

Curt Anker Liliegreen

The Knowledge Centre
for Housing Economics

Jarmers Plads 2
1551 København V
Danmark
Cli@realdania.dk

1. Summary

This paper presents a preliminary sketch of a model for the housing market. Supply and demand relations are formulated for different subgroups, including first time buyers and repeat buyers. As repeat buyers carry out the majority of transactions on the housing market, the personal finances of this group in the form of income, assets and budget constraints play a key role in a model that focuses on actual trades. The personal wealth of a repeat buyer depends greatly on the home equity he can achieve from his existing home. By looking at the home equity for repeat buyers, the paper includes an element where rising house prices can stimulate housing demand. This is not because of speculative expectations created by the price increases but a reflection of the budget restrictions and credit rationing, which earlier hindered the potential repeat buyers in relocating to more expensive dwellings.

The model is characterized by:

- Looking at the supply/demand of owner occupied dwellings as the actual dwellings put up for sale/demanded and not the total housing stock
- Supply and demand are disaggregated into different subgroups of sellers/buyers
- It includes the behaviour of the financial sector through varied degrees of credit rationing
- The price term in the model is to be perceived as a constant quality price index per one quantity unit of housing.
- Not simplifying the model to facilitate the mathematical treatment; instead the mathematical treatment must be adapted to the complexity of the economic model.
- The quantity term of the model is not the number of dwellings, but instead the number of 'housing units', wherein each single dwelling can be described by a certain quantity of 'units' that expresses the size and quality of the dwelling.

The price relation derived from the model is analysed using MATLAB. This work has been carried out by research assistant, Ph.D. student Thomas Trier Bjerring at The Technical University of Denmark, DTU. A stable solution to the system is found and sensitivity analysis performed with regard to changes in parameters.

The first results give an indication that this type of approach to modelling the housing market could be fruitful on an abstract level and as a thought experiment compared to a more traditional approach because it can demonstrate the complexity of the market dynamics – especially the interaction between house prices, credit rationing and capital gains. However it seems also clear that it will be necessary to simplify the model and take a less ambitious approach if such a model should try to explain the actual development.

A model of the type described in this paper could explain the development in the number of first time buyers compared to repeat buyers in the housing market and also explain the development in transactions. This would be of value by itself.

We procure empirical material on the Danish housing market in order to enlighten the distribution of the housing trades by subgroups such as first time buyers, repeat buyers as well as persons who sell their dwellings and leave the housing market entirely. It is possible to gather this information due to the unique Danish CPR register (see www.cpr.dk/in-english), which assigns each individual a unique numerical code, because comprehensive data are tied to this numerical code and by combining this information with the special Danish public BBR system (see www.bbr.dk), a buildings database covering all existing buildings in Denmark. Data for how long repeat buyers occupy their dwellings on average before they relocate to a new dwelling is presented. In section 6, we present data for how many repeat buyers move from a smaller to a larger home ownership dwelling, as well as from a larger to a smaller dwelling. It must be stressed that we are not trying to explain the exact development in the Danish data by our model. It will take a lot more work before this could be achieved.

The data was collected and produced by Master of Science in Economics Stephanie Koefoed Rebbe from the DREAM institution (Danish Rational Economic Agent Model, see www.dreammodel.dk) on the so-called 'research engine' operated by Statistics Denmark at the request of the author. From June 1st 2014, Stephanie Koefoed Rebbe is employed by the consultancy DAMVAD. The Knowledge Centre for Housing Economics and DREAM have been working together on an analysis of the housing market, see[20].

2. The macroeconomic models' focus on the housing market

The macroeconomic models' description of the housing market often takes shape of stock – flow models. These models describe the demand for the overall housing stock, the so-called desired stock. Furthermore, these models describe the supply of housing as being the overall housing stock, which is constant in the short term. From the demand equation and given the constant supply it is possible to derive an equation for housing prices. The adjustment between the desired and actual housing stock occur through the housing price as an error correction model. The actual housing trades are not included in the models. The term 'housing market' as used in the models can be defined as all existing dwellings. To avoid any misunderstandings, in this article we describe the market, where dwellings are actually traded, as 'the market place for housing trades', and not the housing market.

Assessed from the long-term constant quality price index for the housing prices, an error correction model seem intuitively correct, because the index' display a significant long-term trend from where there occur temporary fluctuations. This is true if you consider the renowned Case-Shiller index for the United States in 1890. See [30]. The same is shown in a constant quality price index for Denmark, see [23] and [25].

In macro models such as the Danish ADAM (Annual Danish Aggregate Model), the housing demand is determined by income, the user cost rate, demographic factors as well as the housing price. See [18]. The demand concerns the entire desired housing stock to be matched with the overall housing supply, which is close to unchanged in the short term. A change in demand therefore realise a heavy price reaction in the short term. The price reaction will – via a Tobin's Q correlation, see [34], affect the housing supply and reduce the price fluctuation (the correlation is not without problems. For example, see the working paper 'Rethinking the Housing Model', by Sofie Andersen [4]). The adjustment can last for decades.

Danmarks Nationalbank (the central bank of the kingdom of Denmark) operates the model MONA, wherein the housing market is modelled like in ADAM with certain exceptions. The model is described in [27]. Since the financial crisis, the model's explanation of the housing market has been subject to a thorough analysis, in which the significance of expected housing price increases has been looked at, as well as the effect of whether buyers can perceive the cheapest loans liquidity-wise as the actual housing cost. See [17].

The models does not include supply nor user cost of social public housing, private rentals and cooperative housing, although these housing types represent half of all the dwellings in Denmark.

The models consider all existing dwellings as offered and demanded, but in reality all home owners are not ready to put their dwelling up for sale within a short period of time. For dwellings where the economic preparations necessary for trade can take years, and where the transaction costs are substantial, the adjustment in supply happens slowly. All dwellings are only offered and demanded in the sense that homeowners demand their present dwelling, but this is an abstract concept of supply and demand.

Even those who wish for a quick house sale must contact their banker, mortgage lender or real estate agent and complete a clean up and repair work in the dwelling. Many point out the slow adjustment, although few with greater authority than Robert J. Shiller, see [31] , wherein Shiller states, that 'Real estate markets remain wildly inefficient all over the world. We can only look forward to the day when liquid markets support more trade that might permit something rather closer to the efficient markets that theorists have expected.'

The annual trades make up a fraction of the overall housing stock. The number of traded single family detached homes in relation to all single family detached homes (occupied by the owner) were 5.1% in 2006 and 3.1% in 2011. The trades represent the extreme points in the recent years. Source: Statistics Denmark, www.statistikbanken.dk, table BOL101 among others.

The empirical material supports, that the price adjustment is a process over time. It is common practice, that dwellings for sale are reduced considerably in price during 'the days on market' (the period from where the dwelling is put up for sale until the actual sale). According to Association of Danish Mortgage Banks, the difference between the initial listed prices per m² to the realized selling price were 1.95% in Q4 2005 and rose to 17.54% in Q1 2009. See www.realkreditraadet.dk/Statistikker/Boligmarkedsstatistikken table BM010.

Slow adjustment can also be caused by the seller not accepting a low sales price, because the price will get below the mortgage value of the home ownership. See [32].

According to the Association of Danish Mortgage Banks, single family detached homes had a days-on-market of 104 days at the height of the cyclical period in Q4 2005 and at 204 days in Q3

2013. See Association of Danish Mortgage Banks, www.realkreditraadet.dk/Statistikker/Boligmarkedsstatistikker table BM030.

The empirical material indicates that the housing market does not 'clear' immediately. An imbalance may occur between supply and demand on the market place for housing trades. It is reasonable to assume that the growth rate for housing prices depends on the size of the imbalance.

A deeper understanding of the coherence between housing trades and price may perhaps give a better insight to he short-term fluctuations in the housing prices. See [33] and [5] as well as [2].

On the supply side, the subgroups are:

- Home owners who desire to move to another dwelling. They are called 'repeat buyers' or 'movers' shortened to MVs. In most of the paper the term MVs is used in text and mathematical notation
- Those who desire to leave the real estate market. The abbreviation EXIT is used.
- Speculative operators who offer dwelling to achieve a short-term capital gain. The abbreviation SP is used
- Contractors who supply new dwellings

These groups have different economic conditions, different timeframes and are presumably driven by different preferences. The groups' share of the overall housing trades varies during the business cycle of the housing market. Should you try to estimate the relations on the housing market without including the difference between the shares of the underlying groups, you risk unstable parameters.

On the demand side, the subgroups are:

- First time buyers, typically younger and with financial limitations. The abbreviation FTBs is used in the text and mathematical notation
- MVs buyers who put their dwelling up for sale to finance the purchase of a new dwelling
- SPs who buy with the intent of sublease or sale

The difference between the macroeconomic models' treatment of the housing market and a model using a disaggregated approach appear, when you describe the credit rationing – and other economic incentives and constraints – that drives the various operators. This is especially evident for the younger household that move to larger and more expensive dwellings. At the same time, rising housing price will create an increase in home equity and stimulate the demand from MVs. This is documented by empirical material in section 6.
A disaggregated model may capture some of the situations, where the traditional models could give rise to a lack in parameter stability under a structural break/change. It can be seen as a movement towards microfoundations.

In this paper, we look at the price for traded dwellings, expressed by a constant quality price index of the price per m^2 and the quantity of traded dwellings. With quantity we do not mean the number of dwellings or the value of the dwellings, instead quantity is defined as the dwellings measured in the total number of 'quality units'.

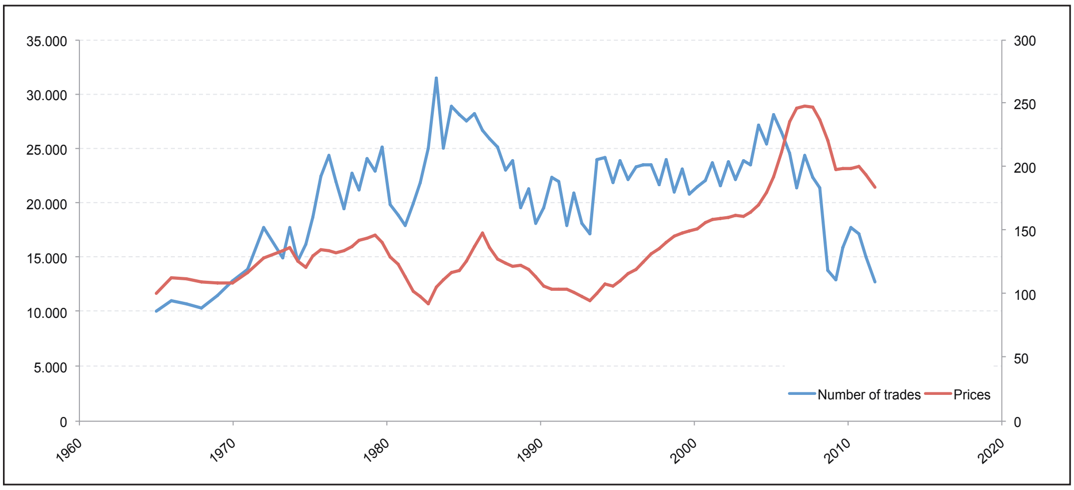
In its simplest form, this is the number of m^2 in the dwelling. In a stringent model, this should be the number m^2, adjusted for quality differences, meaning the dwelling measured as 'the number of quality units'. This is possible as long as you have databases with information regarding every dwelling. Such micro data can be found in the Danish BBR database and includes location, year of construction, remodelling and building materials.

3. The development in housing trades and prices during the business cycle of the housing market

In Denmark, the overall housing stock populated by owner in 2013 was 1.313.000, cf. www.statistikbanken, table BOL101, while the number of completed dwellings by a private developer in comparison was approximately 10.000, see www.statistikbanken.dk, table BYGV50B. Even without considering dwellings that disappear as a consequence of demolitions, the growth rate of the housing stock is only 0.75% annually. In the years when it peaked, the growth was 1.5%.

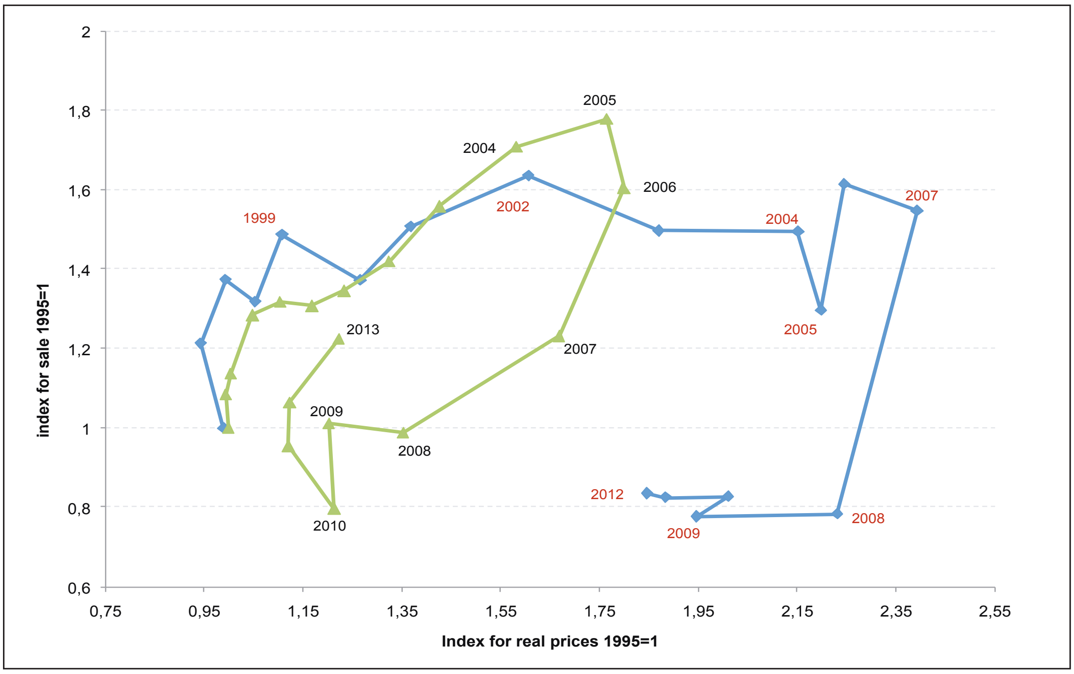
The fluctuations in prices has been considerably larger with nominal increases in Denmark at 25.6% for single family homes from Q2 2005 to Q2 2006 and 29.5% for condominiums from Q1 2005 to Q1 2006., see www.statistikbanken.dk, table EJEN5. This reflects that a short-term increase in demand is not matched by an increase in the housing stock, but leads to increasing housing prices.

FIGURE 1: Number of trades and index of real term housing prices in constant quality for Denmark, 1965-2012



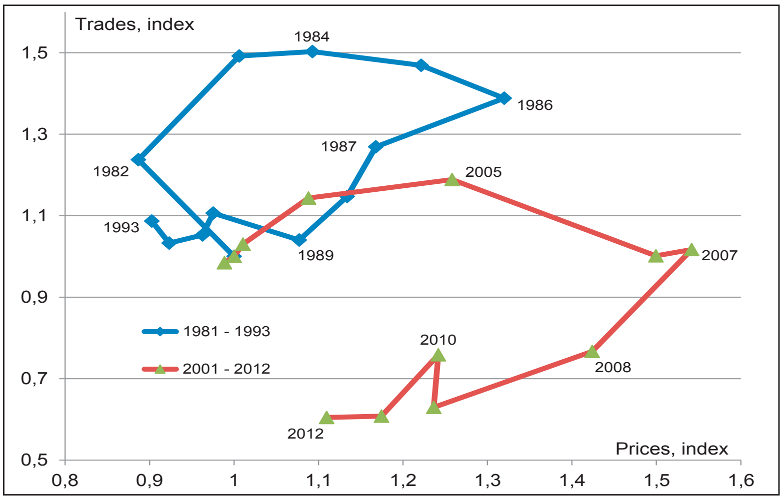
Sources: Number of transaction from The Danish Tax Authorities, real term house prices using the SPAR method (Sales Price Appraisal Ratio) from The Danish Tax Authorities and Statistics Denmark

FIGURE 2: The development in number of housing trades plotted against real term housing price index for the US (green) and England/Wales (blue), 1995-2012/13 shown by index 1995 = 1.00



Sources: S&P /Case Shiller Home Price Indices, National Association of Realtors, US Bureau of the Census, for UK Land Registry, inflation data from IMF WEO October 2013

FIGURE 3: The development in number of housing trades (single family homes) plotted against real term housing prices in constant quality for 1981-1993 and 2001-2002.



(Expressed by index at 1981 = 1 and 2002 = 1)

Sources: Transactions from The Danish Tax Authorities, real term house prices from The Danish Tax Authorities and Statistics Denmark

In figure 1, you see the development in trades together with the development in an index of housing prices (corrected for inflation and in constant quality). A certain context is apparent.

The development in trades as a cyclical indicator primarily attracts interest from mortgage lenders, banks and real estate agents. An exception is the economists who are concerned with the choice between different types of tenure, relocation between tenure types and the formation of new households. In the UK, there is a comprehensive collection of literature and development of models for describing housing need compared to Denmark, see [9].

In figure 2 and 3, trades are plotted against the development in housing prices. In figure 2, we see the price development for dwellings in the US and England/Wales, measured in constant quality and deflated by the consumer price indeces. The constant-quality-price index for the US is Case-Shiller, while the source for England/Wales is the UK Land Registry. In each country, the prices and trades are indexed by 1995 = 1, then the curves for both USA and England/Wales begins in (Y,X) = (1,1). In figure 3, the same type of curves is presented for Denmark for two cyclical periods, 1981-1993 and 2002-2013. Price data here are by the tax authorities and is based on the SPAR method.

The American upturn resulted more in a rise in the number of trades than in housing prices, compared to England and Wales, and Denmark. This suggests limited supply elasticity in England, perhaps as a consequence of planning restrictions, compared to the USA. See [10]. Herein, the supply elasticity is determined, measured as coefficient for the lagged housing prices in the long-term investment relation at 2.014 for the US, 1.206 for Denmark and just 0.395 for the UK.

The development in the figures appears regular until the number of trades reach a minimum during the crisis. Then a more uncertain development can be observed. Characteristically, the trades drop, before the prices drop. The trades only increase marginally in Denmark from 2004 to 2005 and then drop from 2005 to 2006, although the prices increase significantly. In England, the housing crisis started with a sharp decline in trades from 2007 to 2008. In the USA the drop in trades from 2005 to 2007 is significant compared the price drop.

The relationship between trades **Q** and prices **P** in figure 2 for the US can be described as:

$$Q_t = 0{,}651 + 0{,}464 \bullet P_t + 0{,}900 \bullet (P_t - P_{t-1}) + 0{,}749 \bullet (P_{t-1} - P_{t-2})$$

(3,17) (3,00) (2,56) (1,83)

$R^2 = 0{,}82$

t-values are listed in brackets under the coefficients.

A change in the prices from positive to negative may be connected with a substantial change in the trade activity. This is seen both in the UK and in Denmark from 2007 to 2008, and in the US from 2006 to 2007. The drop in trades are in all countries so violent that it suggests the formation of expectations about housing prices play a role in trading activity. There is no unambiguous evidence of, which direction the causality goes. Source [33] leans towards that the causality runs from the trade activity and to the prices.

The circular trajectory which is illustrated in Figures 2 and 3 indicates, that there are several stages in the housing market business cycle. In the following, we give a speculative interpretation, which later is sought supported by empirical data.

In Phase I, the housing prices falls to a bottom level and stabilizes, as it did in Denmark between 1981-1982. The turnaround in the market first appears in an increase in trades, and not in price increases.

In Phase II, both prices and the number of transactions increase, as seen in Denmark in the years 1983 and 1984, as well as the next business cycle in the years 2001 - 2005. In the USA, the development was observed from 1996 to 2005. Meanwhile, the price increases are gaining momentum. Initially, the housing prices are low compared to the long-term trend, measured by a constant-quality price index adjusted for inflation. Therefore, several groups of operators may actively participate in the housing trades: First time buyers are not yet squeezed out by exorbitant prices. Speculators are drawn in by the price increases. The households who want to sell their dwelling to buy another may now carry out this desire, since homeowners who before were technically insolvent now can sell without losses.

In Phase III, the growth rate in prices falls until it reaches zero. Meanwhile, the trades drop as seen in Denmark in the years 1985 and 1986, and again in 2006 and 2007. In the US, this development was seen in 2006. In this phase, the prices are high, and first time buyers may be squeezed out due to credit restrictions applied by banks and mortgage lenders, as well as the households own budget restrictions.

Rational speculative operators would pull out, while the less savvy operators would continue their acquisitions. When the prices have reached a high level, the supply of newly built properties is plentiful as a consequence of a Tobin's Q relation. Constructions begun when the prospects were positive will still be completed, and this increases the supply compared to the demand.

In Phase IV, the prices drop with an increasing rate, and the trades plummet, as seen in Denmark in 1987, 1988, and again in 2008-2012. In the US, this development was seen from 2007 to 2011. The days on market for the dwelling on sale increase. The speculative element is totally gone from the housing marker. New dwellings are still constructed, when the projects have begun in an earlier phase. These dwelling put further pressure on the market.

Phase shifts in the business cycle of the housing market can cause a loss in welfare; a systematic analysis of the damages can be found in IMF Working Paper 08/274 'What happens during recessions, Crunches and busts?', see [12].

This speaks for analysing the phase shifts in the housing marker rather than focussing to the movement towards a long-term point of equilibrium.

4. Transactions and households distributed on types

The operators in both supply and demand are divided into groups by the need for understanding the housing market as a dynamic system. We look at groups, who are presumed to act on the housing market from different incentives, so they will affect the pricing on dwellings in different ways. A change in the relative size of the groups during the business cycle of the housing market may be caused by changes in price, user costs and credit rationing, and a change in group sizes may act back on the prices.

On the demand side, the operators are divided into three groups:

- FTBs are households, who purchase a home, and where none in that household has owned a home within the previous three years.
- MVs.
- Speculative operators (SP).

During 'the housing bubble' in Denmark in the mid-2000s, we saw a tendency that individuals acquired residential properties, often associated with project development, to earn a quick capital gain. The gain could be achieved during the construction phase alone, as price increases peaked at 29% annually for apartments.

In the US, the so-called 'House Flippers' has been responsible for a substantial part of the housing trades. Thus, 'the House Flippers' have accounted for 45% of all new mortgage loans in 2006 in the so-called 'bubble states: California, Nevada, Arizona and Florida according to source [21], see below. The term 'House Flipper' became famous after a reality TV-show 'Flip that house', produced by Discovery Home Channel. The episodes, which initially concerned the housing market in Southern California, aired for the first time in the US in the summer of 2005-2008.

The development in the role of house flippers is described in 'Real Estate Investors, the Leverage Cycle, and the Housing Market Crisis', by Andrew Haughwout, Donghoon Lee, Joseph Tracy and Wilbert van der Klaauw, [21], and 'The Role of House Flippers in a Boom and Bust Real Estate Market' by Jin Man Lee and Jin Wook Choi, [24].

In Europe, we see investors with a longer time horizon, who purchase as a part of a pension plan ('buy-to-let' in the UK) or 'parent purchase' in Denmark. See The Danish Tax Authorities description of the rules at www.skat.dk/SKAT.aspx?oId=1790333.

Homeowners who after selling their home ownership do not move into a new home ownership within the following three years is denoted by the abbreviation EXIT. The group may include seniors, who move from a villa with a garden to a smaller apartment, but in practice it may be a broader group. It may be households who no longer can afford a home ownership and therefore move into a rental property.

On the supply side, you have MVs, see above. They include persons who move from a smaller home ownership to a larger and more expensive home ownership as their income and home equity increase. The group also includes persons who move to a smaller and cheaper home ownership, perhaps due ageing or income loss. In our data, it is possible to screen out those repeat buyers who move from one region to another.

On the supply side, we also count those who have purchased speculatively with the intent of reselling.

The fourth group on the supply side is the contractors that sell newly constructed dwellings. The contractors act from a profit expectation, which can be described with a Tobin's Q relation.
Apart from actual trades, you will to a lesser extent have family transfers as well as individuals who build a house as their own enterprise, either by themselves or as a subcontractor. This phenomenom is less common in Denmark.

It is not necessarily the operators from the same groups, who trade with one another. As it is not the same type of operator that purchase and sell it could have an effect on house prices if the groups have different expectations to the future development of the market. The latter appears to apply according to data collected for Boligøkonomisk Videncenter by Statistics Denmark in 2010 – 2013. During the entire period, we see different expectations among the age groups, as the youngest and the oldest are the most optimistic. See [19].

5. The behaviour of the individual groups

5.1. First time buyers

The number of potential FTBs in the market depends on demographic factors (population in younger age groups and the proportion of single households). The extent to which these potential buyers are able to actualize a purchase depends on housing prices, income, user costs, credit restrictions and transaction costs regarding the trade. The extent to which they actually choose to become buyers is determined by their preferences, user costs and the access to other forms of housing as well as the prices of other consumer goods.

The access to social housing and to the regulated private rental housing with lower rents must in Denmark at the least be considered to be restricted in the growth areas like Copenhagen and Aarhus.

The Danish market is one of the most regulated for private rental housing in Europe, see [11]. DREAM (Danish Rational Economic Agents Model, see www.dreammodel.dk) has calculated, that the rental level in the private rental housing in Copenhagen would increase 100%, if a full liberalization was carried out. See [26]. The low rent implies that it is difficult to access older private rental housing for younger people seeking housing. For public housing with attractive rents there are waiting lists of up to 25 years.

In economic articles the credit restrictions for FTBs are usually requirements to equity and requirements to income, see [33] , as well as [22].

Restrictions are expressed in equation (1) and (2).

(1) $e_t > \alpha \bullet P_t \bullet Q$

– where e is the amount, the buyer must provide as payment, and Pt is the level of housing prices at the time of purchase, and Q represents the desired dwelling expressed in volume units quality. **α** is a coefficient, expressing the required equity as a decimal fraction.

(2) $P_t \bullet Q \bullet U_t < \psi \bullet Y_t$

– where U denotes user cost as a decimal fraction of housing price, **Y** is gross income and **β** is the financial sector's requirements for a maximum debt-servicing ratio, expressed as a decimal fraction.

In Denmark, it is common that the banks offer the rest of the financing from the 80% maximum mortgage and up to 95-100% of the property price. Property purchases also cause relocation costs, legal counselling and more. These costs cannot be financed through a mortgage, but must be covered by savings or other credits. There is therefore a minimum equity requirement associated with house purchase.

The term (2) can be reformulated as:

(2)* $Q \bullet P_t / Y_t < \psi / U_t$

It is known but not recognised publicly that in the financial sector a 'rule of thumb' is applied in form of 'income multiples' or Loan-To-Income ratio (LTI); a multiplier for how much you can borrow compared to your income, and this multiplier historically has been in the range of 3 to 3.5, but that it in the 2000s rose to 4-5. $\boldsymbol{\psi / U_t}$ expresses this multiplier. For banks – but not for mortgage institutes – there is regulation by 'Finanstilsynet', the Danish FSA that customers with a LTI ratio over 3.5 cannot be considered to be first class (having the rating 2a).

The financial sector has argued, that it has only granted IO loans to buyers if they were able to service the same amount of money lended as a traditional loan with instalments. Today, this is a direct claim to the financial sector, according to [14].
If the financial sector complies with this, then (2)* may be rewritten to:

(2)** $Q \bullet P_t / Y_t < \psi / U_t^m$

– where $\mathbf{U^m}$ denotes the maximum user cost at time t, in the form of fixed-rate loans with instalments. The expression $\boldsymbol{\psi / U^m}$ is denoted by the symbol **Ω**. This is a measurement for the credit rationing.

Expression (2)** may appear restrictive by the layman, because IO-loans have become common in Denmark. See [3]. During periods of upswing, the restriction may eliminate a part of the potential buyers in the housing market in areas with high prices. In Denmark, there is a considerable difference between a standard mortgage with fixed instalments and an mortgage without instalments and a variable interest rate, where the interest on a one year 'Flexloan' (F1) at the beginning of 2014 lies at 0.32-0.36%. Therefore, the annual payment before taxes on a one-year adjustable-rate mortgage without instalments at Realkredit Denmark in February 2014 becomes 15.732 kr. for a loan of 1 million kr., while the payment of signing a interest-only fixed rate 3% loan with a 30-year term is 60.648 dkr. the first year.

The credit rationing is discussed in the following section on repeat buyers.
The demand of FTBs can be described as:

$$(3) \quad D^{ftb} = POP^{y} \bullet (k + \Theta \bullet P_t + \Theta \bullet Y_t + \varrho \bullet U_t) \bullet CR_t$$

The expression shows, that the demand is proportional with the share of households in the younger age groups POPy as well as the development of the credit rationing of the financial sector versus first buyers in the form of 'income multiples' denoted by $\mathbf{CR_t}$. The credit rationing works as a 'filter, that lets a certain share of the potential FTBs through, and the efficiency of this filter is measured by **CR** (more precisely by ($\mathbf{1/CR_t}$)). Furthermore, the demand is given by a constant, expressed by k and is a function of housing price P, income Y and user cost rate U. Here applies that:

$$(4) \quad CR_t = (\Omega^1_t / \Omega^{1m})^{\rho}$$

$\mathbf{\Omega_t^1}$ is the credit rationing at time t for FTBs, see section 5.2. $\mathbf{\Omega_t^1}$ is a function of user cost and housing price development. $\mathbf{\Omega_t^1}$ does not immediately react to changes in user cost, but changes with a certain sluggishness.

It is assumed that FTBs adapt the dwelling to their long-term permanent income. They presently accept a high housing cost in their budget and partly assess their economic ability from the cheapest financing of the market liquidity-wise. Their own budget restriction is therefore not the active barrier for their housing demand, when the short-term interest is very low; this barrier consists of the credit rationing. An easing of the credit rationing has a huge effect on demand. The effect – however – is decreasing, the larger the credit multiplier $\mathbf{\Omega_t^1}$ becomes, because the households own budget restriction and preferences for other consumer goods begins to weigh in. This is expressed by the coefficient $\boldsymbol{\rho}$.

The development in the number of FTBs during the period 2000-2009 is shown in table 1.

5.2 *Repeat buyers/movers*

Homeowners, who desire to move to a more expensive dwelling, is subject to budget limitations and credit rationing by the financial sector. This implies, that they must remain in their present dwelling for a number of years while repaying their debt and wait for a capital gain from increasing housing prices.
It makes no sense to move to a dwelling of equal value as the present (unless you relocate to another region due to a new job, education or similar). We assume, that a certain quality boost is necessary for the household to move due to the transaction costs and other costs such as information gathering and solving practical problems. This boost is denoted by **k**. When the desired quality boost is lesser than k, it may be met through improvement works in the dwelling.

$$(5) \quad Q^1 \bullet P_t \bullet k \leq Q^2 \bullet P_t$$

– where $\mathbf{Q^1}$ is the previous dwelling expressed in quantity units and $\mathbf{Q^2}$ is the new dwelling. In the following, the inequality sign is substituted with an equal sign, and it is assumed, that those who are moving up in the movement chain on average move to homes that are k times more expensive.

A movement transaction release movement costs $\mathbf{Q^1 \bullet P_t \bullet \Lambda_1}$ on the sale and $\mathbf{Q^2 \bullet P_t \bullet \Lambda_2}$ on the purchase.

For those who have purchased a dwelling with quality quantity **Q** at time **t-i**, the payment $\boldsymbol{\alpha}$ and mortgaged with interest-only loans, it applies, at they at time t have a home equity of:

$$(6) \quad e_t = Q \bullet (P_t - (1-\alpha) \bullet P_{t-i} - P_t \bullet (\Lambda_1 + k \bullet \Lambda_2))$$

$\mathbf{P_t}$ is related to $\mathbf{P_{t-i}}$ through the price increase from time **t-i** to time **t**. This rate is rarely constant from year on year, but for the sake of simplicity it is initially described as an annual nominal increase rate $\boldsymbol{\varpi}$. The growth is thereby a function of the number of years (dwelling seniority), in which the household resides in the dwelling:

$$(6)^* \quad e_t = Q \bullet (P^1_{t-i} \bullet ((1+\varpi)^i \bullet (1-\Lambda_1-\Lambda_2 \bullet k)-1-\alpha))$$

The size of the home equity depends on the value of the acquisition cost of the property, the rate of price increases, the number of years the owner lived in the dwelling, the disbursement rate and the trade costs. This assumes that there are no mortgages in the home equity.
When the income multiplier restriction is in force, then the maximum price the household may be approved of is:

Purchase price (maximum)

$$\Omega^2_t \bullet Y_t + e_t = \Omega^2_t \bullet Y_{t-i} \bullet (1+\phi)^i + Q \bullet P^1_{t-i} \bullet ((1+\varpi)^i \bullet (1-\Lambda_1-\Lambda_2 \bullet k)-1-\alpha)$$

The annual increase rate in income **Y** is set at $\boldsymbol{\phi}$. To the household, it is crucial, how much more expensive the new dwelling can be in relation to the present dwelling. This ratio is a 'strength indicator' (IND) for its demand. We assume that the first dwelling was bought at the maximum price the household could be approved for at the time of purchase. This dwelling's price has since increased. The strength indicator must assume values above **k**, before the household is interested in moving. A certain share of households will realise an increase in housing consumption but stay in their present dwelling due to improvements, rebuilding or extensions.

$\boldsymbol{\Omega^1}$ is the credit multiplier for FTBs and $\boldsymbol{\Omega^2}$ the multiplier for repeat buyers.

$$IND_t = (\Omega^2_t \bullet Yt + e_t)/(\Omega^1_{t-i} \bullet Yt_{-i} \bullet (1+\varpi)^i)$$

which can be written as

$$IND_t = (\Omega^2_t/\Omega^1_{t-i}) \bullet ((1+\phi)/(1+\varpi))^i + ((1+\varpi)^i \bullet (1-\lambda_1-\lambda 2 \bullet k)-1+\alpha)/(1+\varpi)^i$$

The expression $\boldsymbol{\Omega^2_t/\Omega^1_{t-1}}$ can be re-written as $\boldsymbol{(\Omega^2_t/\Omega^2_{t-i})(\Omega^2_{t-i}/\Omega^1_{t-i})}$, where it partly express a time factor – that the credit evaluation is eased or tightened over time, and partly express a seniority factor, that a more lenient evaluation of older households than younger first time buyers occur. (7) can therefore be written as:

$$IND_t = (1/(1+\varpi)^i) \bullet ((\Omega^2_t/\Omega^2_{t-i}) \bullet (\Omega^2_{t-i}/\Omega^1_{t-i}) \bullet (1+\phi)^i - (1-\alpha)) + 1-\lambda_1-\lambda_2 \bullet k$$

(7)* says, that the possibility to move up in the movement chain is affected positively by the development in income, and by the fact that the credit multiplier is higher for second time buyers than for first time buyers. The transaction costs and the mortgaging ratio **(1-α)** at the first purchase affect the indicator negatively. Isolated, an increase in housing prices does not affect the strength indicator positively. A capital gain for the household strengthens their equity, but the desired new dwelling increases in price at the same time. The factor $\boldsymbol{1/(1+\varpi)^i}$ decreases over time due to increasing housing prices. But the housing price increases cause the credit rationing to ease, and this affects the indicator.

If the credit rationing term in form of payment demands given by **(1)** is tight for all buyer groups, then an increased home equity will have a larger significance. One must therefore assume, that a self-reinforcing element is added to the housing price increases.

For MVs an increase in housing prices becomes a signal for an increase in transactions, where the existing dwellings the repeat buyers supply to the market is an equivalent to the Tobin's Q effect on the investments. As the MVs are credit rationed, to them a 'Tobin's Q' cannot only depend on prices, but also on a series of technical factors as described in (7).
Expression (7) shows that hardly any fast skips in the value for **k** happens, unless the credit multiplier changes. It is therefore important to model the behaviour of the financial sector in order to catch the short-term movements in the housing prices.

FTB are assumed to start off with a small equity capital, and it is therefore restriction (1) that limits their purchase. As the household gains an increase in home equity, it becomes restriction (2) in form of 'income multiples', which limits the home equity. The household has a long-term desired housing quantity(the households 'housing dream'), which is aligned with its expected long-term income. A difference between the households and the financial sector's perception of which dwelling the household can afford may occur. The buyer's perception of how many funds he has available may be based on the cheapest loans liquidity-wise in the market. This is obvious, because previously the real estates agents displayed the costs of financing based upon such loans (This has since changed due to the so-called Formidlingsbekendtgørelse, see [13]).

The implications of the interest-only loans with a variable interest are expressed in the so-called Rangvid report (by the chairman, Professor Jesper Rangvid, CBS): The lower payment may also cause those buyers who are short-sighted, irrational or who do not fully understand the reason for the lower payment to take on more debt than they would otherwise'. See [15].

In its extreme form, the difference between the credit multiplier accepted by the bank and the one found reasonable by the household is expressed by the difference between user cost with the cheapest financing on the market ($\mathbf{U^{min}}$) and the maximum user cost requiring 'standard financing' (a misleading name in recent years), $\mathbf{U^m}$. The households beliefs in their own purchasing options in relation to the bank are:

$$(U_t^m/U_t^{min}) \bullet \Omega_t \bullet Y_t + e_t$$

The question is whether the bank or mortgage lender always lets Um form the basis for the lending. In Denmark, the financial sector stated itself, that this was always the case. Today it has become a direct regulation, see [14]. Several conditions, however, can encourage the bank to ease their credit policy. If the bank believe the housing prices to be increasing, it makes sense to ease the credit rationing, because the security in the mortgage will improve, see [6] and [7].

From 1990 and until the end of 2004, the interest for ordinary and special mortgage bonds showed a gradually declining trend, cf. www.statistikbanken.dk, table MPK3. The financial sector could believe a regime change in monetary policy had taken place with the formation of the EMU and the turmoil in the economy due to the globalization. In the years before the crisis, these conditions played a role in the economic debate, cf. for example 'Globalisation and OECD Consumer Price Inflation' by Nigel Pain, Isabell Koske and Marte Sollie [29].

A third condition is the competition between the different parties of the financial sector. This is amply illustrated in the 'The Financial Crisis Inquiry Report', Final report of the National Commission on the Causes of the Financial and Economic Crisis in the United States, January 2011 [16], especially pp 87 – 98, that the aggressive competition from Ameriquest, New Century and Countrywide caused a more lenient credit evaluation in the US.

The Danish mortgage model has historically shown its robustness, and an exogenous shock to the economy should be particularly powerful before it can shake the mortgage system. This is documented in a series of analyses and stress test, for example see [32] and [28]. This could have given the financial sector a sense of false security.

Historically, the non-payment of mortgage loans in Denmark has been related to severe macroeconomic crisis. See [1]. This analysis of major recessions in Denmark since 1816 shows that they on average occurred every 37 years. Large downturns in GDP go so far back in history, that the financial sector hardly was aware of the risk.

The mortgage institutes yearly net lending to the owner occupied sector in Denmark is described in www.statistikbanken.dk, table MPK7. The net loans rose from 62.9 billion Danish kroner in 2004 to 115.1 billion Danish kroner in 2005 and 105.7 billion Danish kroner in 2006. During the first ten months of 2013, the net loans were only 13.6 billion Danish kroner. The net lending has been very sensitive to changes in house prices during the boom and bust cycle. The present lending activity is by the lay man and by the popular press often seen as a 'credit squeeze'.

The behaviour of the financial sector in relation to credit rationing is described as:

$$(8) \qquad \Omega_t^2 = \Omega_0^2 + h \bullet ((\psi/U_t)-(\psi/U^m))$$

In the relation for **Ω^1**, **h** is replaced by **h***, and **h*<h**

h and **h*** is either **0**, or replaces a value equal to a decimal fraction between 0 and 1. h and h* are different from **0**, when two conditions are met simultaneously:

1. Condition **$P' > 0$** for a certain period.
2. Condition **$U_t + \tau < U^m$** **τ** is set at a fixed value of 0.01

When user cost has dropped to a low level for a long time, and additionally house prices are increasing, the financial sector will be easing credit rationing gradually.

We have tried to replace the expression (8) with the following equation:

$$\Omega_t^2=\Omega_0^2 \text{ if } P'\leq 0$$

$$\Omega_t^2=\Omega_{t-1}^2+h\bullet((\psi/U_t)-\Omega_{t-1}^2)$$

– and the same type of formulation for **Ω^1** where h is substituted with **h***

We expected this to result in a more stable solution. The system was solved in this alternative version but the result was highly unstable. Even small changes in parameters resulted in major changes in the development of **P_t**. This was the case when parameters such as **Ω^1, Ω^2, ρ, e** and **Λ** were changed. We see this as a typical example of stiffness in differential equations and of complexity.

The demand and supply function for repeat buyers is described as:

$$(9) \quad D_t^{mv} = (POP^{mv}/POP)\bullet K_t\bullet(e^*+f^*\bullet IND_t-r^*\bullet U_t)$$

$$(10) \quad S_t^{mv}= (POP^{mv}/POP)\bullet K_t\bullet(e+f\bullet IND_t-r\bullet U_t)$$

The supply and demand are proportional with the share of homeowners in those age groups, where relocation is frequent and primarily occur to a another home ownership. Here, it is approximated by households with the oldest member under 60 years. At the same time, supply and demand are proportional to the size of the housing stock in terms of quality units. In addition, supply and demand is determined by a constant (some will always relocate eg. due to demographic events), and is a function of the strength indicator and user cost. **e, f** and **s** are not similar to **e *, f *** and **r ***. It applies that **f *> f**.

The development in the number of MVs is shown in table 1 (the difference between the overall number of trades and the number of FTBs). The table shows that repeat buyers in the Capital city rose from 54% in 2000 to 62%, when the housing prices peaked in 2006.

5.3 *Individuals who leave the housing market*

A number of people leave the housing market permanently in any period as a consequence of demographic events – illness, disability and death, or because they want to move to and remain in another form of dwelling such as a senior home or a small rental apartment near the city center. In Denmark, these matters have been investigated by among others Boligøkonomisk Videncenter in a survey, called 'Ældres boligforhold – En undersøgelse af boligforholdene for personer mellem 70 og 90 år', [8].

The number of seniors leaving the housing market is assumed to be proportional to the number of individuals in the respective age groups, but it is

also affected by the price level. In a liquid market with high prices, it is assumed to be tempting to 'take the profit' and leave the market.
The supply of dwellings created by these groups can therefore be described as:

(11) $S_t^{ex} = (POP_t^o/POP_t) \bullet K_t \bullet (a + b \bullet P_t^{\epsilon})$

Relation (11) states that the supply is directly proportional to the quantity of the housing stock owned by the senior population multiplied by an expression that shows supply as a function of housing prices. There is also a constant in relation (11), because there will always be a certain number of households who leaves the market for owner occupied dwellings due to factors such as illness or death.

Table 3 shows the development in EXITs in the period 2000-2009. The table shows that the number of EXITs follow house prices, which especially apply to individuals above 60 years of age. In 2005, where the number of transactions peaked, there were 18.981 transactions, where the sellers left the owner occupied market. In 2009, there were 14.829 EXIT trades. In 2005 8.888 households with the oldest member over 60 years of age left the owner occupied market, this figure was reduced to 6.415 households in 2009.

5.4 *Speculative operators*

Private individuals' and smaller companies' acquisitions of home ownerships with the intent of a quick resale at a profit is not altogether uncommon in Denmark. This is, however, a well-known phenomenon in the United States, often associated with a slight refurbishment of the property. In their article 'The role of House Flippers in a Boom and Bust Real Estate Market', see [24] Lee and Choi, argues that House Flippers accounted for 18.2% of all trades in single-family homes in Chicago in the period from 1995 to 2010. They define House Flippers as individuals who owned the dwelling for less than two years. The authors conclude that the flipper-activity culminated in the period from 2004 to 2006.

In 'Real Estate Investors, the Leverage Cycle, and the Housing Market Crisis' , see [21], it is argued that in the 'Bubble States' (Florida, California, Arizona and Nevada) investors (individuals owning more than one property) accounted for close to 45% of all mortgages, concerning acquisitions of properties (Figure 3c, page 42) in 2006 and 2007. Individuals owning two properties accounted for 25 percentage points. The share of individuals owning four or more properties rose significantly from 2005 to 2006.

As the speculators built their property portfolio, they 'remove' their net properties from the housing market and stimulate price increases. When the properties are removed from the housing market for a longer period of time and rented out, no properties are removed net from the housing market, but it is assumed that the properties are vacant during the days on market period. Greatly simplified, the speculators' supply and demand is described as:

(12) $D^{sp} = \pi \bullet (\delta \bullet P_t' + \varepsilon \bullet P_t'')$

(13) $S^{sp} = \pi \bullet \beta \bullet P_t'$

π is a dummy variable, which shall be valid:

$\pi = 0$, when $P' - \Lambda < \mu$

$\pi = 1$, when $P' - \Lambda \geq \mu$

μ is the risk premium. It is set to 0,025, but is upped to 0,05, when the housing price is significantly higher than the long-term average. It is assumed that the speculators keep the property for such a short period, so the alternative return does not influence their decisions. The speculators are not active in the market before the housing price increases surpasses a threshold. Then they act in proportion to the rate of increase in prices.
At the same time, it applies to the speculators' demand and supply that:

(14) $\int_t^{t+n} S(t)dt < \int_t^{t+n} D(t)dt$

6. *Empirical material*

Table 1 shows that first time buyers in Metropolitan area of Copenhagen accounted for an absolute increasing number of trades until and including 2005. When the housing prices peaked in 2006, the FTBs dropped in absolute number and measured by their share of the overall number of trades, they were positioned at 38.5% versus 48.4% in 2000. In the provinces, first time buyer form a relatively smaller share of the buyers in all those years. We also see a tendency that the FTBs are pressed when the housing prices are peaking, but to a lesser extent than in the Metropolitan area of Copenhagen.

TABLE 1: The number of first time buyers (as in number of trades) and share of the overall number of trades in the Capital and the rest on the country 2000-2009

YEAR	FTB CAPITAL REGION	ALL TRADES CAPITAL REGION	FTB IN % IN CAPITAL REGION	FTB IN REST OF COUNTRY	ALL TRADES IN REST OF COUNTRY	FTB IN % IN REST OF COUNTRY
2009	5.664	13.884	40,80	12.494	33.816	36,95
2008	6.194	15.327	40,41	16.656	44.589	37,35
2007	7.875	19.188	41,04	20.984	55.162	38,04
2006	7.993	20.882	38,28	23.343	59.479	39,25
2005	10.772	25.729	41,87	25.822	63.206	40,85
2004	10.310	24.712	41,72	23.834	57.731	41,29
2003	8.949	20.780	43,07	22.702	52.327	43,38
2002	9.050	20.519	44,11	21.847	49.167	44,43
2001	9.584	21.371	44,85	21.527	48.513	44,37
2000	9.943	21.734	45,75	22.511	49.641	45,35

FIGURE 4: The number of years in home ownership for repeat buyer before moving to a new home ownership, distributed by age groups, 1993-2010, in the Capital.

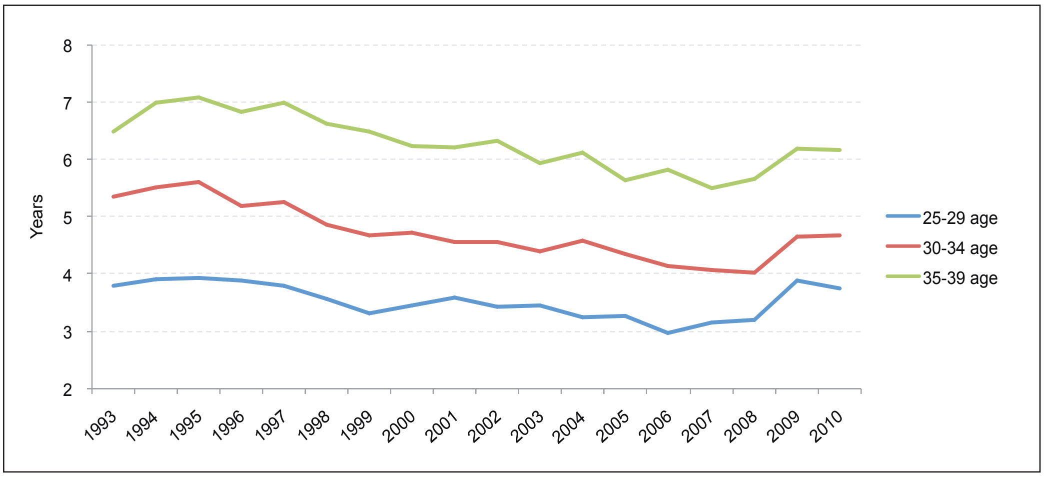


FIGURE 5: The number of years in home ownership for repeat buyer before moving to a new home ownership, distributed by age groups, the provinces, 1993-2010.

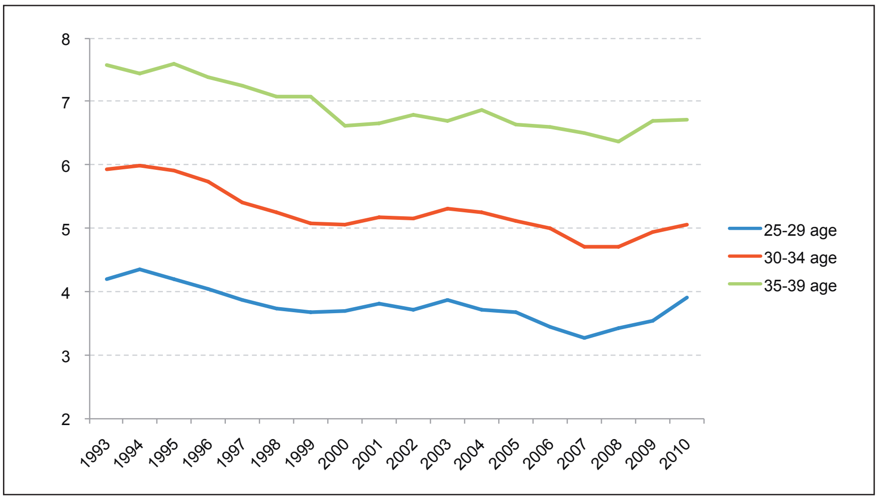


Figure 4 and 5 shows that while the FTBs are pressed when the housing prices are peaking, the situation is easier for repeat buyers. On average, the younger repeat buyers reside in their existing dwelling for a shorter period of 1-1½ years. It shows, that the increasing housing prices are not a problem to them, even if they move to a larger and more expensive dwelling. This is explained by the fact that they can take advantage of any achieved capital gain, and/or that the credit rationing is eased more for them than for the FTBs. This is in compliance with the relation, established above for supply and demand from repeat buyers.

The number of EXIT's follows the housing prices. This apply to the provinces in particular, where the number of EXITs above 60 years was doubled from 2003 to 2005, but since has dropped to the starting point. These data are in compliance with the relation, established for the supply from the EXIT group.

TABLE 2: The number of repeat buyers that move to a dwelling, which is larger, smaller or of same size as their previous 1994-2009

YEAR	MOVING TO LARGER	MOVING TO SMALLER	MOVING TO SAME SIZE
2009	4.319	2.676	503
2008	7.222	4.325	772
2007	8.791	5.499	688
2006	9.333	5.628	683
2005	9.001	5.454	443
2004	8.064	5.017	555
2003	6.721	4.438	546
2002	6.239	4.429	543
2001	6.607	4.734	603
2000	7.008	4.891	696
1999	7.471	4.977	550
1998	8.046	5.117	599
1997	8.045	5.325	644
1996	8.061	5.085	524
1995	7.604	4.828	754
1994	7.347	4.467	487

From 1994 to 2009, 206.359 relocations occurred by MVs within all the individual regions together, and of these 58.1% took place to larger dwellings, while 37.3% moved to smaller dwellings. For all the years it apply that those who move to a larger dwelling upgraded to more m^2 than those who moved to a smaller dwelling went down in size. For example, in 2009 4.319 moved to a larger dwelling and increased their home with 57.6 m^2 on average, while 2.676 moved to a smaller dwelling and on average they reduced the size of their home with 48,8 m^2. The difference between individuals who move to larger and smaller dwellings respectively and the difference in number of m^2 involved in the movement overall implies, at a significant additional consumption of dwelling units occur due to trades made by repeat buyers.

TABLE 3: The number of individuals leaving the housing market for a minimum of three years distributed on the Capital and the rest of the country 2000-2009

YEAR	EXIT IN CAPITAL REGION	EXIT OVER 60 YEAR IN CAPITAL REGION	EXIT IN REST OF THE COUNTRY	EXIT OVER 60 YEAR IN REST OF THE COUNTRY
2009	3.522	1.100	11.307	5.315
2008	3.786	1.385	13.155	6.426
2007	3.777	1.447	12.367	6.186
2006	4.780	1.775	13.914	7.030
2005	5.002	1.812	13.979	7.076
2004	4.255	1.586	13.141	6.654
2003	4.219	1.437	12.678	6.171
2002	4.294	1.372	12.335	5.748
2001	4.280	1.311	11.810	5.539
2000	4.411	1.357	12.407	5.906

Relations in the disaggregated housing market model

The model consists of three main relations, one for supply, one for demand and a relation for the adjustment in the prices. The supply relation is (18), the demand relation (19) and the adjustment relation for the prices are (20). When the demand is larger than the supply, the supply will make up the traded quantity. When the demand is larger than the supply, then the demand will make up the traded quantity

The speculative operators are only active in certain situations. There we apply a dummy variable, which can assume the value 0 or 1, so they activate when the price increases reach a certain threshold.

Restrictions for the coefficients apply, where some appear as dummy variables, cf. the above mentioned.

(15)

$$S_t = S_t^{mv} + S_t^{ex} + S_t^{sp} + I_t = (10) + (11) + (13) + I_t$$

The supply **S** is the sum of the supply from repeat buyers (**mv**), from individuals leaving the housing market (**ex**), from speculators supply of properties (**sp**) they previously have taken over as well as newly constructed dwellings (I).

(16) $$D_t = D_t^{ftb} + D_t^{mv} + D_t^{sp} = (3) + (9) + (12)$$

(17) $$I_t = c \bullet (P_t/C_t)^{\ni}$$

The investments are given by a Tobin's Q relation. Both investments and housing stock K is expressed in quantity units. The housing stock is in discrete time given by:

(18) $$K_t = K_{t-1} \bullet (1-\vartheta) + I_t$$

(19) $$P'_t = d \bullet (D_t - S_t) + g_0 \bullet D_t + g_1 \bullet (D_t - D_{t-1})$$

For the last two parts of (19) D is used, when $D_t < S_t$, and S, when $D_t > S_t$.
The price increase depends of the size of the inequality between supply and demand and of the absolute number of trades as well as the change in number in number of trades.

When expression (15), (16) and (17) is applied in (19), and **P'** is drawn from the left side of the equals sign to the right, a 2nd-order non-linear differential equation emerges.

7.2 List of expressions used in the relations

Technical coefficient/elasticity and constant terms
1 a = share of households with homeownership that leaves the market of non-economic reasons, **0<a<1**

2 b = the seniors' supply elasticity with regard to price, **b> 0**

3 c = the investors' supply elasticity for new construction in regard to Tobin's Q, **c > 0**

4 d = the effect on the growth rate of the housing prices of an imbalance between supply and demand in the housing market. **d > 0**

5 e = share of homeowner, who independent of economic factors desire to move to another home ownership dwelling, **0 < e < 1**

e* denotes the reverse constant term for the demand for new home ownerships for the established homeowners.

6 f = elasticity for the homeowners' supply of dwellings on sale in relation to the so-called strength indicator IND.

With **f*** the elasticity in the demand relation is denoted **f*>f**

7 g = the price increase rate's sensitivity to the composition of the demand **g > 0**

8 h = expresses the adjustment rate in the credit multiplier **Ω2** , when a drop in user cost occur.

0 < h < 1. h is thought to be in the range of **0, 1 – 0,33**

With **h*** the adjustment rate in denoted towards first time buyers **h*< h**

9 α = demand to equity at housing purchases set to 0,05 as a starting point, which means 5%.

10 ß = elasticity for speculator activity in regard to the price increase rate. **ß > 0**

For practical reason, we set **β = 0,9•δ**

11 ρ = elasticity for effect of changes in the credit rationing Ω for the first time buyers' demand

12 δ = elasticity for speculative operators' activity prompted by price increases, **δ > 0**

13 ʎ = overall transaction costs at both purchases and sales = 0,06 as starting point

14 Θ = elasticity for first time buyers' housing demand by income, **Θ > 0**

15 ϴ = elasticity for first time buyers' demand of housing prices level **ϴ < 0**

16 ϱ = elasticity for first time buyers' demand of user cost, **ϱ < 0**

17 ε = elasticity for speculative operators' activity prompted by changes in the price increase rate (2. derived by the prices) **ε > 0**.

18 π = dummy variable for speculative trading. Assumes values **0** or **1**.

19 µ = risk premium for the speculative agents. This premium is as a starting point set equal to 0,025, but can increase to 0,05 when $P_t - P^m > 1{,}5$, where P^m is the long-term average (here in the comparison adjusted for inflation).

20 ψ = the financial sectors restriction with regard to the maximum debt servicing ratio. (As a starting point assumed to be 0,36 in relation to available income and 0,18 in relation to the gross income expressed by Y_t)

21 Ω = credit rationing by the financial sector, maximum price for housing purchase divided by household income. Distributed on first time buyers = **Ω¹** and repeat buyers = **Ω²**.

We operate with the initial values Ω_{t0}^{1}**=3,5** and Ω_{t0}^{2}**=3,85**. **Ω** develops over time as a function of **U**.
22 τ = the significant change in user cost; is this threshold violated, the financial sector will react. **τ** is set at 0,01 of practical reasons.

23 ϑ = abandonment of dwellings. Set at 0,01.

24 Є = the price elasticity for seniors' supply of dwellings, when they desire to leave the owner occupied sector.

25 э = exponent in the expression for Tobins Q, **э>1**

26 k = constant term in first time buyers' demand

27 r = coefficient for user cost in supply for movers

r* express coefficient to user cost in demand for movers

Economic variables

28 S = supply of dwellings measured by quality units

29 D = demand of dwellings measured by quality units

30 P = price per quality unit in dwellings, with **P'** as 1. derived and **P"** as 2. derived

31 I = investments in new dwellings, expressed by the quantity of quality units

32 K = the stock of dwellings expressed by the quantity of quality units

33 IND = indicator for the homeowners' possibility for move to a more expensive dwelling

34 CR = the degree of credit rationing for first time buyers

35 C = index for construction costs

36 Y = the development in the household income

37 U = user cost.

U^{m} denotes the long-term average for user costs for traditional mortgaged loan with instalments

U_{t} denotes the actual user cost.

Demographic variables and characteristics

38 ftb = used as superscript denotes first time buyers

39 ex = = used as superscript denotes exit, households that leave the housing market

40 mv = = used as superscript denotes movers, repeat-buyers

41 POP = population, all households in dwellings

42 POP^{y} **=** households without homeownership who potentially demand a home ownership. **Y** is short for 'young'. This group forms the basis for those who actually demand a dwelling as FTBs measured by D^{ftb}. As proxy to POP^{y}, we use the number of households of younger non-homeowners.

43 POP^{m} **=** households in home ownerships who potentially could move to another home ownership. m is short for 'middle-aged'. The group form the basis for those who actually demand a new dwelling, measured by D^{mv}. As proxy for POP^{mv}, we use the number of households containing the eldest person above 60.

44 POP^{o} **=** households in home ownerships who potentially desire to leave the housing market. o is short for 'old'. This group form the basis for those who put their dwelling up for sale and leaves the housing market after sale, measured by S^{ex}. As proxy for POP^{o}, we use the number of households containing the eldest person above 60.

Exogenous variables	3	U,Y,C
Endogenous variables	9	P, I, K, S^{ex}, S^{sp}, S^{mv}, D^{ftb}, D^{mv}, D^{sp}
Parameters	31	a,b,c,d,e,e*,f,f*,g,h,h*,α, β,ρ,δ,ʎ,Θ,ϴ,ϙ,ε,μ,ψ,Ω1, Ω2,τ,ϑ,k, Є,r,r*,э
Calculated expression	4	IND, CR, D, S
Dummy variables	1	π
Demographic factors	4	POP, POP^{o},POP^{y},POP^{m}
Altogether	52	

8. The behaviour of the system

Method

When equation (17), (18) and (19) are placed in relation (21) a second order differential equation emerges with the prices $\mathbf{P_t}$ and the first and second order derivative $\mathbf{P'_t}$ and $\mathbf{P''_t}$. The equation is non-linear. The equation can be expressed as:

$$(22)\quad P''_t = C1 \bullet P_t + C2 \bullet P'_t + C3 \bullet P_t^{C4} + k$$

An equation of this kind can only be dealt with through numerical analysis. The coefficients **C1**, **C2**, **C3** and **k** are rather complicated as each of them consist of many parameters. The model is also using binary dummy variables. It was deemed likely that the equation suffers from the phenomenon called 'stiffness'. Some of the parameters in the model like **Ω** are not constant but varies over time depending of the value of usercost and is influenced by $\mathbf{P'_t}$ through a dummy variable.

The model is only a sketch and we do not know the values of most of the parameters in the model. We are at present only able to make 'guesstimates' about the values of these parameters. In this early attempt it is sufficient to see if the model can be solved at all. In order to do so, we use our guesstimates and the model is reformulated in to a set of first order differential equations. If a solution can be obtained, we will perform sensitivity analysis of the effect of minor changes in key parameters and see if the model behaves logically.

We attempt to solve the model using the 'backward differentiating formula'. This method is especially well suited when problems with stiffness can occur. Because of the dummy variables a spatial branch-and-bound algorithm is used to eliminate systematically portions of the decision space that cannot contain the solution.

The software tool MATLAB is applied, and hypothetical values of the exogenous variables income, usercost and construction costs are chosen together with 'guesstimates' for the value of the parameters. There is not space enough in this article to list these values but they can be sent upon request.

Equation (22) is rewritten. $\mathbf{P''_t}$ is substituted with $\mathbf{dy_1/dt\ (y'_1)}$ and $\mathbf{P'_t}$ is substituted with $\mathbf{y_2}$.

$$y'_1 = y_2$$

$$y'_2 = C1 \bullet y_1 + C2 \bullet y_2 + C3 \bullet y_1^{c4} + k$$

The solution for house prices takes the form in figure 7. This should only be considered as proof that the system can be solved, and the curve serves as a benchmark we will use in the sensitivity analysis. We have 40 time periods in the figure.

Development in endogenous and exogenous variables

FIGURE 6 : Chosen values of exogenous variables – usercost

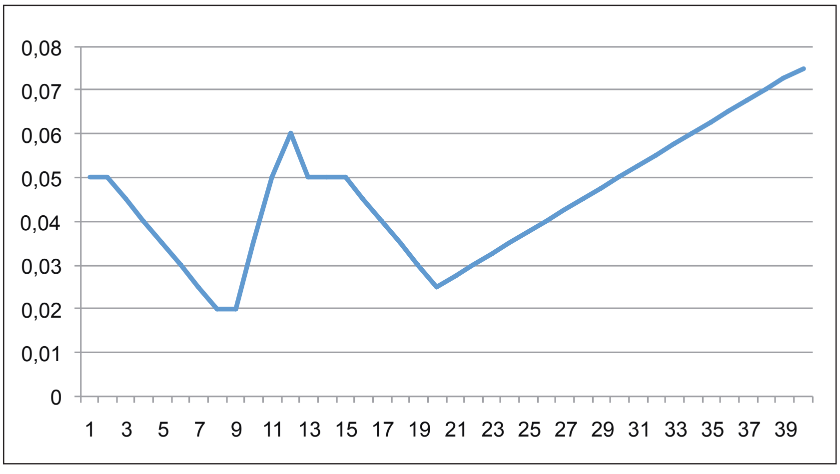


FIGURE 7 : Chosen values for income and construction costs

The values are chosen in order to test how the model reacts to an exogenous shock with fluctuations in usercost. We also want to see the effect of a long term increase in income of the model. Besides this, it must be stressed, that the values are totally hypothetical. We are not trying to test the model against the factual development because this is an early stage and we consider the model immature.

FIGURE 8 : The solution to the system, development in house prices

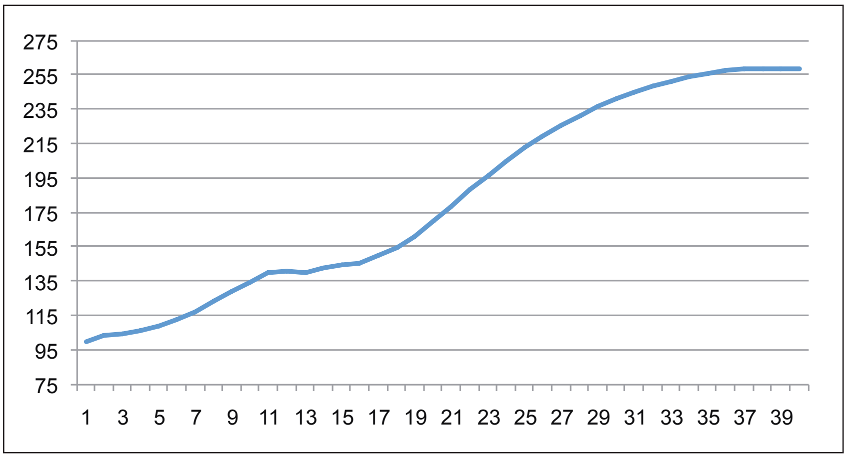
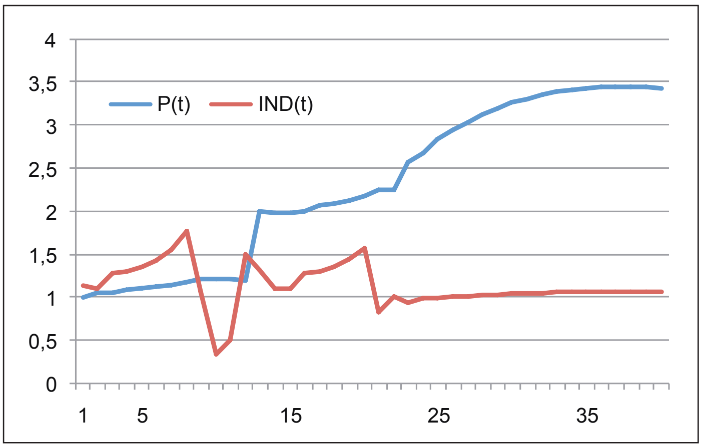


FIGURE 9: Development in strength indicator IND(t) and house prices P(t), model version where ρ=0,04 (price index divided by 100)



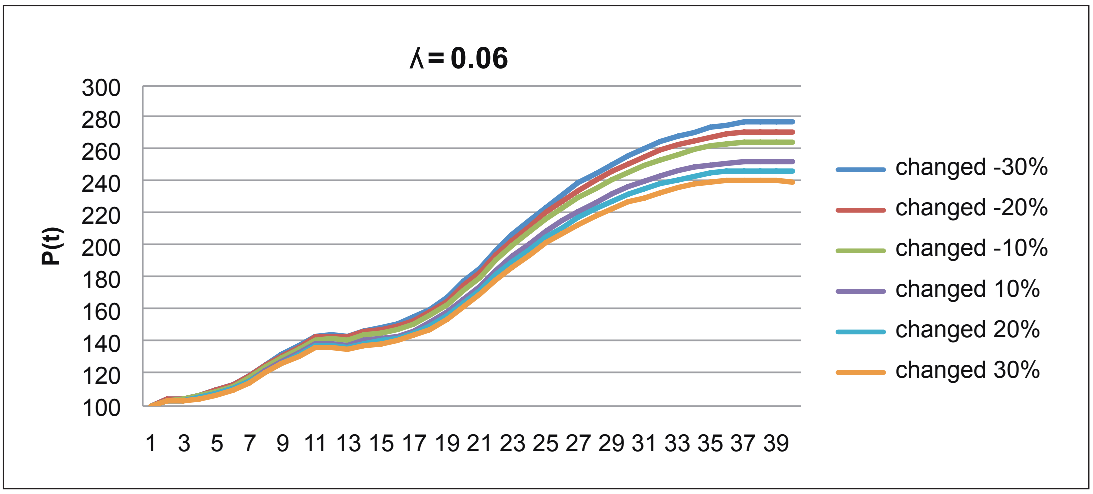
The solution to the equation is **P(t)**. It can only be concluded that it is possible to solve a system of this nature but not much can be interpreted from the form of the curve. It seems at a first glance that both the complex factor **IND(t)** and **P(t)** can obtain meaningful values but the real test of the solution is an analysis of how it will behave when a sensitivity analysis is performed with regard to the many different parameters.

It will take up to much space to perform a sensitivity analysis for all parameters in the model. Therefore we will only show data for a few parameters that demonstrate how a model of this lay out could work.

Sensitivity analysis

The first example of the sensitivity analysis shows the effect of changes in the transaction costs **ʎ**. They are set at 0,06, and the effect on house prices are analysed for changes in the range of -30% to 30% , that is from 0,042 to 0,078. The effects are shown in figure 7.

FIGURE 10: The development in house prices under different transaction costs



The house prices at the end of the period varies from index 239 to index value 277. This is a minor variation where prices are 16% higher when the transaction costs are at the low end of the range compared to the maximum costs and not too much should be put in to these preliminary findings.

A different example is the effect when one than one parameter is allowed to change value. In figure 6 the value of the parameter **ρ** is set to 0,4. If this parameter is set to 0,04 the fluctuations in the value of the indicator $\mathbf{CR}_t$ will almost disappear.

We will now look at the effect of changing the values of the parameters **h** and **h***. These parameters appear in relation (8) and determines how quickly the financial sector changes credit rationing **Ω** when usercost change. A bigger **h** means a quicker response and should impact prices positively.

FIGURE 11: effect of changes in h and h* when ρ is fixed at 0,04 and h=0,33, h*=0,15 before changes

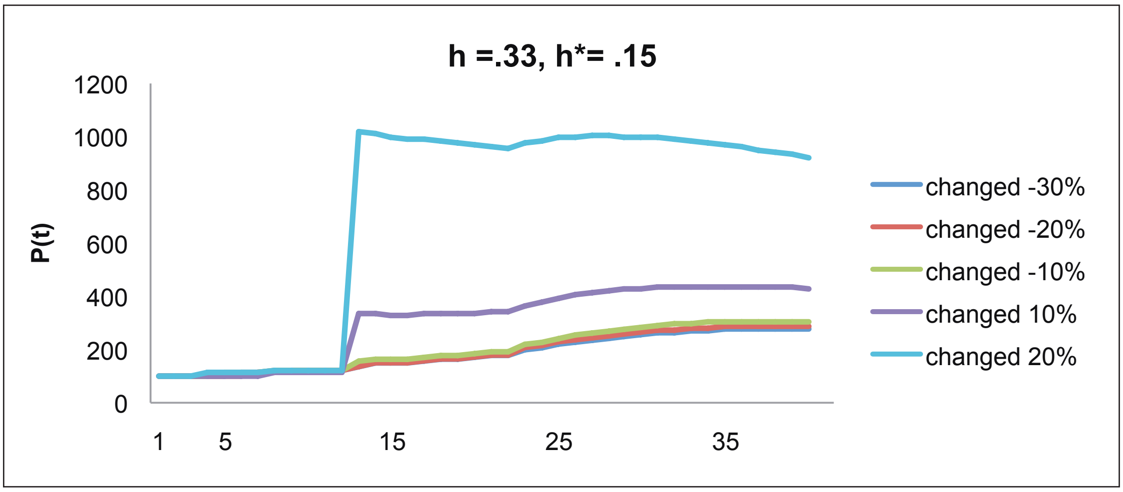
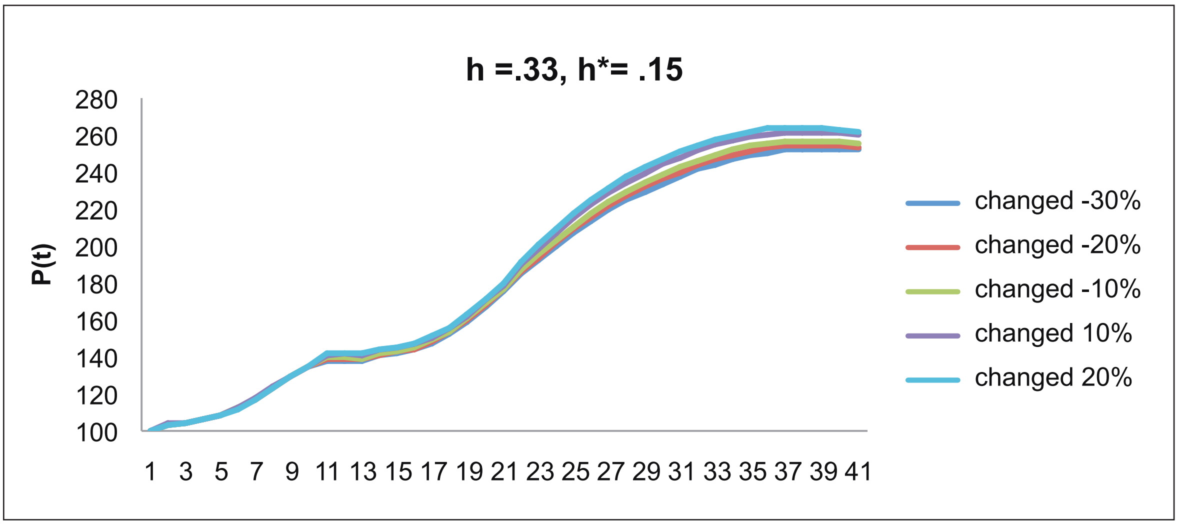


FIGURE 12: effect of changes in h and h* when ρ is fixed at 0,4 and h=0,33, h*=0,15 before changes.



It is evident that the parameters in the model effect each other. When the effect of credit rationing for FTBs is low due to **ρ** being close to 0, there is a very strong reaction to changes in **h** and **h***. It seems obvious that the value of both **h**, **h*** and **ρ** and their effect on the price fluctuations should be carefully analyzed in the future work.

The result gives an indication of a high degree of complexity in house price dynamics. If this result can be substantiated in the future work with the model it can serve as another word of caution against relying too much on historical data and econometrics when explaining the housing market unless these methods have a solid foundation in an understanding of the structural conditions in the market (microfoundations).

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